A Bit not as a Unit of Information - A Qubit is not a Unit of Quantum Information

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ABSTRACT

While Shannon introduced "binary digits, or more briefly *bits*" as units of information, a binary digit (and, thus, a bit) originally and still means a character of "0" or "1", which may be represented by a binary symbol or storage cell. As a result, a bit often means a binary symbol or storage cell, which is inherited by a "qubit", a quantum bit, introduced by Schumacher as "the fundamental units of quantum information". Schumacher himself uses a bit, not a qubit, as a unit of quantum information and a qubit is a quantum binary symbol or storage cell.

Keywords - **binary digit, bit, quantum bit, qubit.**

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I. INTRODUCTION

There are some confusions on meanings of a bit. Though Shannon introduced "binary digits, or more briefly *bits*" [1] as units of information stating: " The choice of a logarithmic base corresponds to the choice of a unit for measuring information. If the base 2 is used the resulting units may be called binary digits, or more briefly bits, a word suggested by J. W. Tukey." [1], a bit often means a binary symbol or storage cell into which a bit of information is encoded/stored.

For example, start and stop bits of start stop synchronization methods such as RS232C are symbols with fixed values to establish synchronization between senders and receivers and do not carry any information. As start bits have evolved to be training symbols of OFDM communications such as Wifi, the start bits are symbols. A "flag bit" is an example of a bit used as a binary storage cell. That ASCII is a 7-bit character code means an ASCII character is represented by 7 binary symbols or storage cells.

For more examples, IJANA papers [2, 3, 4] contains "bit" in its title not as a unit of information but as a binary symbol or storage cell. "bit sequences" of [2] implies a bit is a countable entity. "64-Bit" of [3] means 64 binary symbols are transmitted in parallel. "Bit Error Rate" of [4] means a probability of a bit as a symbol is wrongly decoded.

However, as is exemplified by a statement "The bit is the most basic unit of information in computing and digital communication. The name is a portmanteau of binary digit.[1] The bit represents a logical state with one of two possible values." in [5], two meanings are not properly distinguished. A bit which "represents a logical state with one of two possible values" is not a unit of information but a binary symbol or storage cell.

This paper investigates and discusses origin and validity of such usages. In section II, original meanings of "binary digit" are discussed. In section III, meanings of a "bit" and a "binary digit" in Shannon's original paper [1] are investigated. In sections IV, similar confusions on meanings of a "qubit" in Schumacher's original paper [6] are investigated and discussed. Section V concludes the paper.

II. ORIGINAL MEANINGS OF A "BINARY DIGIT"

Obviously, a "digit" means a decimal digit, a character of "0", "1", "2", "3", "4", "5", "6", "7", "8" or "9" of ASCII characters for which an ASCII code of 30, 31, 32, 33, 34, 35, 36, 37, 38 or 39 is assigned respectively, which may be represented by 7 binary symbols or storage cells.

Similarly, the original meaning of a "binary digit" is a character of "0" or "1", which may be represented by a binary symbol or storage cell. As Shannon introduced "bit" as "binary digits, or more briefly *bits*" [1], a "bit" also has the same meaning.

A bit as a binary symbol or storage cell is a countable noun whereas amount of information is not countable. Though exactly N bits of information can be encoded or stored into N bits of binary symbols or storage cells, with noise, which is considered in PART II of [1], less information can be decoded or restored from them.

III. MEANINGS OF A "BIT" AND A "BINARY DIGIT" IN SHANNON'S ORIGINAL PAPER

In [1], the word "bit" appears 34 times. All of them are used as a unit of information.

However, the phrase "binary digit" also appears in [1] 20 times and, except for the first one of "binary digits, or more briefly *bits*", they are used not as a unit of information but as a character or a symbol. For example, in the following two usages of "If the channel sequences are not already sequences of binary digits" and "its first binary digit will be 0, otherwise 1.", a binary digit obviously means a character "0" or "1". In another usage of "Thus we can approximate a

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coding system to encode messages from this source into binary digits with an average of $\frac{7}{4}$ binary digit per symbol.", binary digits are something into which information is encoded. That is, Shannon himself uses a binary digit as a binary symbol.

IV. MEANINGS OF A "QUBIT" IN SCHUMACHER'S ORIGINAL PAPER

A word "qubit" was first proposed in [6] by Schumacher in 1995 as "quantum bits, or "qubits," are the fundamental units of quantum information" [6]. Moreover, the same or similar statements are still found in more recent articles such as [7]. However, as is stated "In quantum systems, however, the expression for entropy (first proposed by von Neumann [3]) is not identical to the Shannon entropy." in [6], quantum information or von Neumann entropy was introduced in [8] by von Neumann in 1955, 40 years before [6]. As such, if a "qubit" were "the fundamental unit" of quantum information, quantum information had been used without unit for 40 years.

[6] is on quantum version of source coding theorem stated as "the von Neumann entropy S(ρ) of an ensemble is just the mean number of qubits necessary to encode the states in the ensemble in an ideal coding scheme" [6], but, it should be noted that, in the theorem, "qubits" are countable entities.

Actually, just as a unit of quantum energy is J (Joule), not quJ, a unit of quantum information is not qubit but a "bit" or, with natural logarithm used in [8], "nat" ($1\text{ nat} \fallingdotseq 1.44\text{ bits}$).

Moreover, in [6], Schumacher himself uses "bit" as a unit of quantum information stating" The von Neumann entropy of ρ will equal the Shannon entropy of the message source only in the special case when the signals $|a_M\rangle$ are orthogonal to one another, in which case the signal states are eigenstates of ρ. If the signals are not orthogonal, then S(ρ)<H(A), and the eigenstates of ρ may have no simple relation to the signal states". As von Neumann entropy "S(ρ)" and Shannon entropy "H(A)" are compared as "The von Neumann entropy of ρ will equal the Shannon entropy" and "S(ρ)<H(A)", they must share the same unit of "bit".

From the statement "This is accomplished by replacing the classical idea of a binary digit with a quantum two-state system, such as the spin of an electron." [6], it is obvious that "the spin of an electron" is a countable physical entity usable as a quantum binary symbol or storage cell and is not a unit of information. The statement of "The qubit will be our fundamental unit of quantum information, and all signals will be encoded into sequences of qubits." [6] is similarly confused as [5]. Latter half the statement means qubits are something into which information is encoded. That is, a quantum bit or qubit is a quantum binary symbol or storage cell.

V. CONCLUSION

It is clarified that though Shannon introduced a "bit" as a briefer notation of a "binary digit" and uses them as a unit of information, "binary digit" (and, thus, "bit") also have a meaning of a character of "0" or "1" represented by a binary symbol or storage cell and are used to mean a binary symbol or storage cell even by Shannon himself.

It is also clarified that though Schumacher introduced a "qubit" as if it were the unit of quantum information, a "qubit" is a quantum binary symbol or storage cell and Schumacher himself uses a bit, not a qubit, as a unit of quantum information.

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Biography

Masataka Ohta received B.S. in Computer Science in 1982 and M.S. in 1984 from the University of Tokyo and Ph.D. on "High Quality Computer Graphics" in 1994 from Tokyo Institute of Technology. He is currently a Lecturer of Institute of Science Tokyo (the university was renamed from Tokyo Institute of Technology in 2024). His primary research is on Internet technologies at various layers and he is an author of rfcs 1554, 1815 and 1995. His research at the physical layer is on photonics including fully buffered optical packet switching with many wavelength (thus, high speed) packets requiring short fiber delay lines.